Nuclear Interaction From Effective Field Theory

Honours Final Presentation

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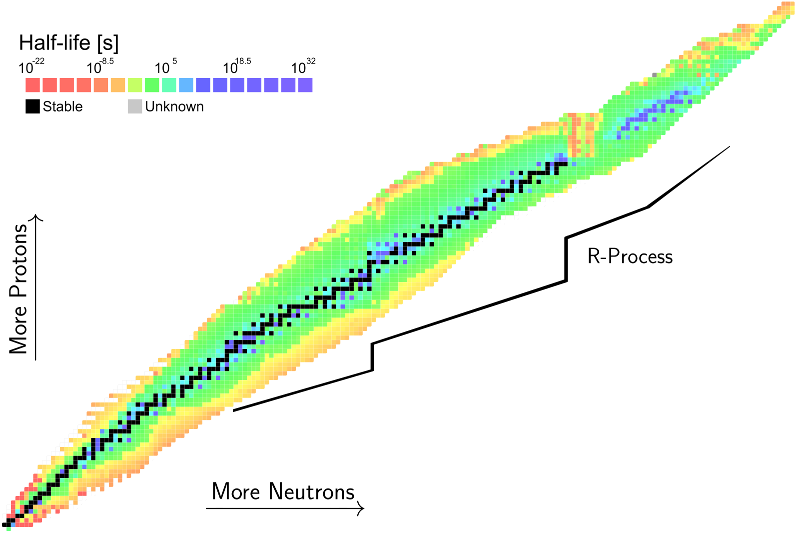
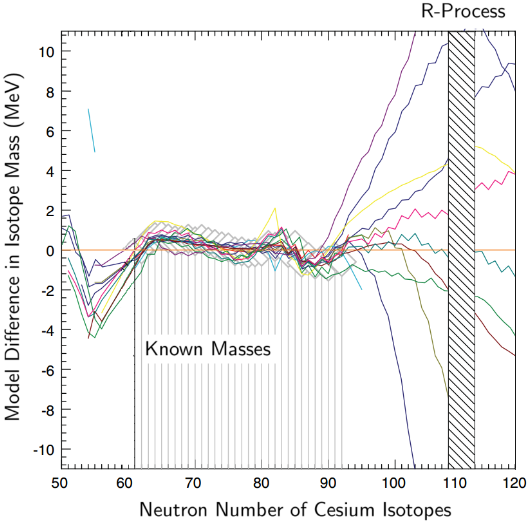
Supervisor: Cédric Simenel

The Nuclear Chart



Credit: Edward Simpson

Masses of Cesium Isotopes



Adapted from Blaum, Phys Rep **425:1** (2006)

The Nuclear Many-Body Problem

- Three ingredients are needed:
  - Nucleon interaction properties
  - A way to solve the resultant many-body problem
  - Experimental data

The Nuclear Many-Body Problem

more phenomenological →

Lattice QCD	Chiral Perturbation Theory	Relativistic Mean-Fields	Non-Relativistic Energy Density Functionals
1 nucleon	2–16 nucleons	Structure of even nuclei	Structure and dynamics of all nuclei
1 GeV	700 MeV	100 MeV	10 MeV

Non-Relativistic Approaches

- Uses an Energy Density Functional (EDF) to describe nucleon interactions
- Needs lots of parameters
- Hence lots of parametrisations

The Goal

- By studying the origin of nuclear forces in higher energy physics, can we derive constraints on the non-relativistic EDF?

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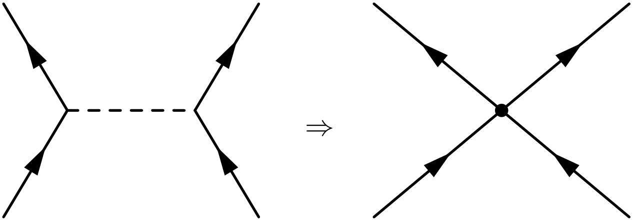
- By studying the origin of nuclear forces in higher energy physics, can we derive constraints on the non-relativistic EDF?
- At low energies, describe interactions using effective field theory

Developing A Toy Model

- Model with two particles
  - The nucleon N
  - The sigma σ, a scalar meson



Developing A Toy Model

- The Lagrangian

$$\mathcal{L} = \overline{N}(i\gamma^\mu \partial_\mu - M)N + \frac{1}{2}(\partial\sigma)^2 - \frac{m^2}{2}\sigma^2 - g\sigma\overline{N}N$$

- Use path integral to quantise theory

$$Z = \int DN \; D\overline{N} \; D\sigma \; e^{i\int dx^4 \mathcal{L}[\sigma, N, \overline{N}]}$$

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- Assume $E_{\text{nuclear}} \ll m$ and expand

Developing A Toy Model

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- The low-energy effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \overline{N}(i\gamma^\mu\partial_\mu - M)N + \frac{g^2}{2m^2}(\overline{N}N)^2 + \frac{g^2}{2m^4}(\overline{N}N)\partial^2(\overline{N}N) + O(m^{-6})$$

Developing A Toy Model

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- From this we can calculate low-energy properties of nuclear systems

$$\mathcal{E}_{\text{INM}} = C\rho^{5/3} - \frac{g^2}{2m^2}\rho^2$$

Credit: http://hubblesite.org/newscenter/archive/releases/2002/24/image/a/

Developing A Toy Model

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- We need extend to more realistic theories

Credit: http://hubblesite.org/newscenter/archive/releases/2002/24/image/a/

Challenge 1: Many Mesons

Meson	Spin$^{\text{Parity}}$	Isospin	Strangeness	Mass (MeV)
π	0^-	1	0	135
K	0^-	1/2	1	494
η	0^-	0	0	548
η'	0^-	0	0	958
ρ	1^-	1	0	775
ω	1^-	0	0	782
K^*	1^-	1/2	1	892
ϕ	1^-	0	0	1019
$f_0(500)$	0^+	0	0	500
$f_0(980)$	0^+	0	0	980
$a_0(980)$	0^+	1	0	980

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The Chosen Mesons

- The $f_0(500)$ or σ, isoscalar-scalar field σ
- The pion, isovector-pseudoscalar field

$$\pi_a = \begin{pmatrix} \pi_+ \\ \pi_0 \\ \pi_- \end{pmatrix}$$

- The omega, isoscalar-vector field

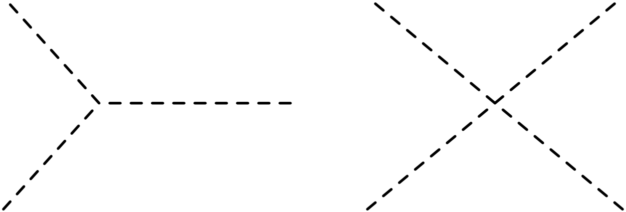
$$\omega^\mu = \begin{pmatrix} \omega^0 \\ \omega^1 \\ \omega^2 \\ \omega^3 \end{pmatrix}$$

- The rho, isovector-vector field ρ_a^μ

Challenge 2: Meson-Meson Interactions

- Need to include self-interaction terms like

$$\mathcal{L}_{\text{sigma}} = \frac{\mu}{3!}\sigma^3 + \frac{\lambda}{4!}\sigma^4$$



- Unless μ and λ are small, difficult to quantitatively study

Challenge 3: Many Interactions

Renormalisable					Non-Renormalisable
Order 3	Order 4				Order 5
σ^3	σ^4				σ^5
$\omega^2\sigma$	$\omega^2\sigma^2$	$(\omega^2)^2$	$(\omega_\mu\rho_a^\mu)^2$		$\sigma(\partial\sigma)^2$
$\rho^2\sigma$	$\rho^2\sigma^2$	$\rho^2\omega^2$	$(\rho^2)^2$	$(\rho_a^\mu\pi_a)^2$	$\sigma(\partial^\mu\omega_\mu)^2$
$\pi^2\sigma$	$\pi^2\sigma^2$	$\pi^2\omega^2$	$\pi^2\rho^2$	$(\pi^2)^2$	...

Problem Solving

- Expand at low energies
- The low energy coupling constants cannot be computed from high-energy physics
- But they can be fitted to nuclear data

$$g(\alpha_0, \alpha_1, \alpha_2, ...)(\overline{N}N)^2 \rightarrow g(\overline{N}N)^2$$

Problem Solving

$$\mathcal{L}(N, \overline{N}, \sigma, \omega, \rho, \pi) \rightarrow \mathcal{E}(\rho, \tau, J_{ij}, ...)$$

- Our model incorporates
  - Symmetries
  - Energy Scales
  - Meson interaction channels
  - Existence of non-linear interactions
- But our results are insensitive to details of the mesonic physics

The Low-Energy Energy Density Functional

$$
\begin{aligned}
\mathcal{E} = \frac{1}{4m^2}\int dx^3 \Big[&(-3g^2+3\lambda^2-3\mu^2)\rho^2 + (g^2-\lambda^2+5\mu^2)\rho_a^2 \\
&+ \frac{1}{2M^2}\left[\left(-5g^2+5\lambda^2(1-2\mu_\omega)+3\mu^2(1-2\mu_\rho)\right)\right]\rho\nabla\cdot\vec{J}_v \\
&+ \frac{1}{2M^2}\left[\left(-g^2+\lambda^2(1-2\mu_\omega)+3\mu^2(1-2\mu_\rho)\right)\right]\rho_a\nabla\cdot\vec{J}_{va} \\
&+ \frac{1}{16m^2}\left(-56d_1+24d_2-\frac{m^2}{M^2}(6\alpha^2+(\mu_\omega^2-4\mu_\omega+8)\lambda^2+3(\mu_\rho^2-4\mu_\rho+8)\mu^2)\right)\rho\Delta\rho \\
&+ \frac{1}{16m^2}\left(8d_1-72d_2+\frac{m^2}{M^2}(2\alpha^2-(\mu_\omega^2-4\mu_\omega+8)\lambda^2+(\mu_\rho^2-4\mu_\rho+8)\mu^2)\right)\rho_a\Delta\rho_a \\
&+ \frac{1}{4M^2}\left((\mu_\omega^2-4\mu_\omega-12)\lambda^2+3(\mu_\rho-2)^2\mu^2+6\alpha^2-\frac{8M^2}{m^2}(d_1+3d_2)\right)\rho\tau \\
&+ \frac{1}{4M^2}\left((\mu_\omega-2)^2\lambda^2-(\mu_\rho^2-4\mu_\rho-20)\mu^2-2\alpha^2-\frac{8M^2}{m^2}(d_1-d_2)\right)\rho_a\tau_a \\
&+ \frac{1}{4M^2}\left(\frac{8M^2}{m^2}(d_1+3d_2)+6\alpha^2-\lambda^2(5\mu_\omega^2-20\mu_\omega+16)-3\mu^2(5\mu_\rho^2-20\mu_\rho+16)\right)J_{ij}J_{ij} \\
&+ \frac{1}{4M^2}\left(\frac{8M^2}{m^2}(d_1-d_2)-2\alpha^2-\lambda^2(5\mu_\omega^2-20\mu_\omega+16)+\mu^2(5\mu_\rho^2-20\mu_\rho+16)\right)J_{ija}J_{ija} \\
&+ \frac{1}{8M^2}\left(6\alpha^2-3\lambda^2(\mu_\omega-2)^2-9\mu^2(\mu_\rho-2)^2\right)\left(J_{ij}J_{ji}+J_s^2\right) \\
&+ \frac{1}{8M^2}\left(-2\alpha^2-3\lambda^2(\mu_\omega-2)^2+3\mu^2(\mu_\rho-2)^2\right)\left(J_{ija}J_{jia}+(J_{sa})^2\right)\Big].
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\end{aligned}
$$

Inequalities

- We will focus on the central and spin-orbit terms:

$$\mathcal{E} = \int dx^3 \sum_{t=0,1} \left(C_t^\rho \rho_t^2 + C_t^{SO} \rho_t \nabla \cdot \vec{J}_{vt} + \ldots \right)$$

$$-\tfrac{1}{3} C_0^\rho \leq C_1^\rho \quad (1) \qquad 5 C_1^{SO} \leq C_0^{SO} \quad (2) \qquad C_0^{SO} \leq C_1^{SO} \quad (3)$$

$$C_0^{SO} \leq -\frac{5}{8} \left| \frac{3}{5} C_0^\rho + C_1^\rho \right| + \frac{3}{8}(1 - 2\mu_\rho) \left(\frac{1}{3} C_0^\rho + C_1^\rho \right) \quad (4)$$

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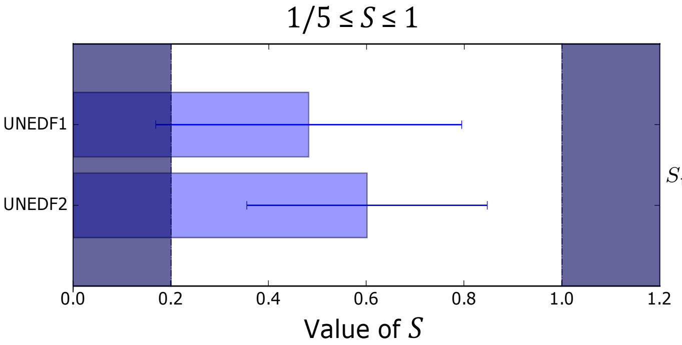
Inequalities

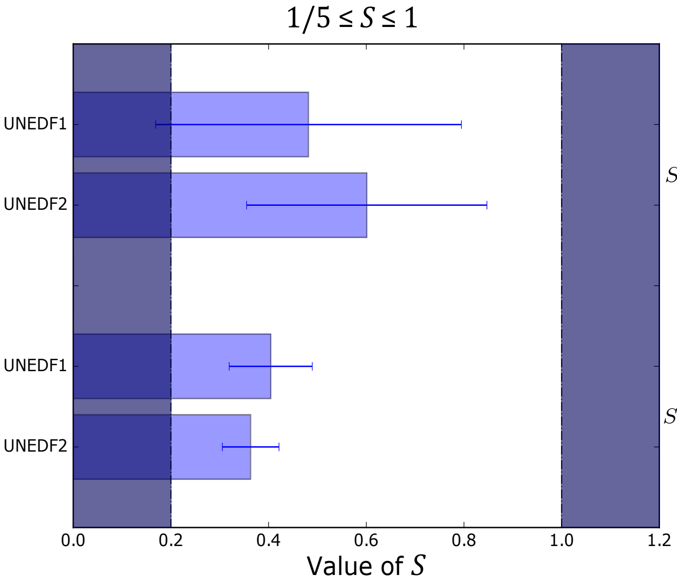
Name	Year	(1)	(2)	(3)	(4)	(5)
UNEDF2	2014	✓	✓	✓	✓	✓
UNEDF1	2012	✓	✓	✓	✓	✓
UNEDF0	2010	✓	X	✓	✓	X
SLy4	1997	X	✓	✓	✓	✓
SLy6	1997	X	✓	✓	✓	✓
SLy10	1997	X	✓	✓	✓	✓

Isospin Dependence of the Spin-Orbit



$$S_{\text{fit}} = \frac{C_1^{SO}}{C_0^{SO}}$$

Isospin Dependence of the Spin-Orbit



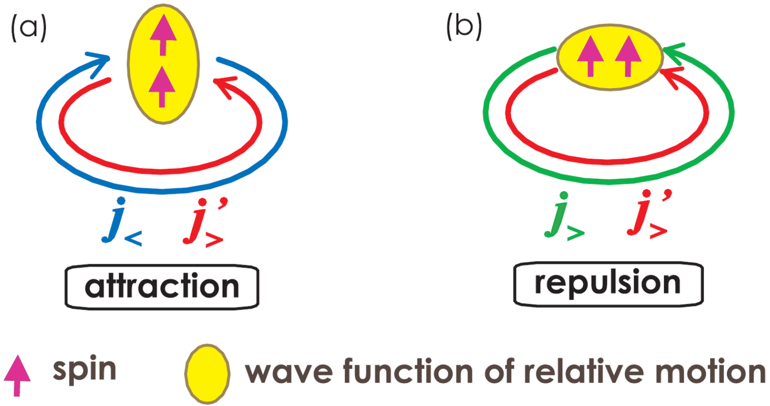
$$S_{\text{fit}} = \frac{C_1^{SO}}{C_0^{SO}}$$

$$S_{\text{calc}} = \frac{1}{5} + \frac{(1-2\mu_\rho)C_0^\rho}{10C_0^{SO}} + \frac{3(1-2\mu_\rho)C_1^\rho}{10C_0^{SO}}$$

Conclusion

- Using basic properties of high energy physics, nuclear interactions can be partially constrained
- More sophisticated models have limited utility

Future Work: Tensor Terms



From Otsuka *et al*, Phys Rev Lett **95:232502** (2005)

Future Work: Spin Densities

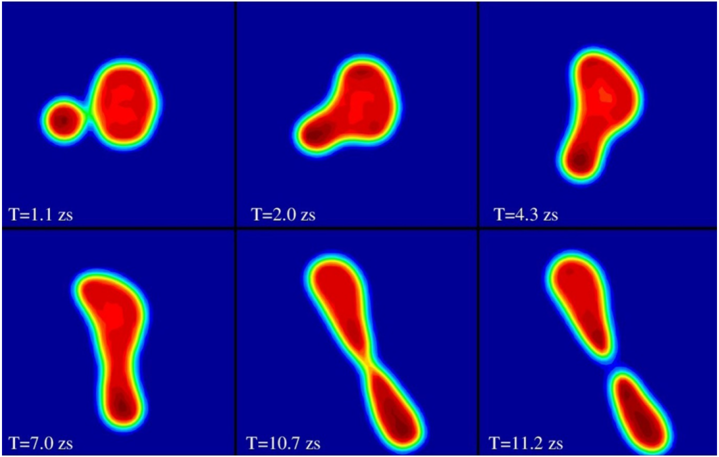


Figure: Quasifission of ^{48}Ca+^{249}Bk

From Umar, *et al*, Phys Rev C **92:024621** (2015)